3-5

Solving Two-Step Inequalities

Essential question: How do you solve inequalities that involve multiple operations?

CC.7.EE.4b

1 EXPLORE  Solving Two-Step Inequalities

As a salesperson, you are paid $52 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

A Write an inequality to represent the number of sales you need in order to be paid at least $100 for the week.

B Method 1: Solve the equation by covering up the term with the variable.

Cover the term containing the variable. Think: "Some number plus 52 is at least 100."

What number plus 52 is at least 100?

Now uncover the term.

Think: 3 times some number is at least 48.

3 times ____ equals 48.

C Method 2: Solve the equation by undoing the operations.

Step 1: Make a table.

<table>
<thead>
<tr>
<th>Operations in the Inequality</th>
<th>To Solve</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. First x is _____________ by 3.</td>
<td>1. First _________ 52 from both sides of the equation.</td>
</tr>
<tr>
<td>2. Then, 52 is ____________</td>
<td>2. Then _________ both sides by 3.</td>
</tr>
</tbody>
</table>

Then, starting with the last operation in the inequality write the opposite of the step. Continue writing the opposite until every step is accounted for.
**Step 2:** Apply the steps in the “to solve” column to solve the inequality.

\[ 3x + 52 \geq 100 \]

\[ 3x \geq 48 \]

\[ x \geq \]

You must make at least _____ sales.

**REFLECT**

1a. How can you check your solution?

1b. Would the graph of the solution set be a ray or individual points? Explain your answer.

**EXAMPLE  Solving Two-Step Inequalities Containing Fractions**

Solve \( \frac{x}{-4} - 5 < -2 \). Then graph the solution set.

A  Complete the table and solution steps.

<table>
<thead>
<tr>
<th>Operations in the Inequality</th>
<th>To Solve</th>
<th>Solution</th>
</tr>
</thead>
</table>
| 1. First \( x \) is \( \underline{\text{___________}} \) | 1. First \( \underline{\text{___________}} \) to both sides of the inequality. | \( \frac{x}{-4} - 5 < -2 \)
| 2. Then, \( \underline{\text{___________}} \) | 2. Then \( \underline{\text{___________}} \) both sides by \( \underline{\text{___________}} \) and reverse the inequality symbol. | \( \frac{x}{-4} \)

\( \frac{x}{-4} = -5 < -2 \)

\( \frac{x}{-4} = -5 \)

\( x = 20 \)

\( x > \)

B  Graph the solution on a number line. Put an _____ circle on -12, since the inequality sign is greater than, not greater than or equal to. Then, the ray goes to the _______.

B  Graph the solution on a number line. Put an _____ circle on -12, since the inequality sign is greater than, not greater than or equal to. Then, the ray goes to the _______.

x >
TRY THIS!

Solve each inequality.

2a. \(-13 > \frac{x}{8} - 3\)  
2b. \(40 \leq -3x + 10\)  
2c. \(-\frac{x}{3} + 5 < -10\)

REFLECT

2d. How is solving inequalities different from solving equations?

EXAMPLE  Solving Real-World Inequalities

Cathy has $100 saved to spend on clothes. She wants to purchase a winter jacket for $40 and some sweaters that cost $20 each. How many sweaters can Cathy buy?

A  Write an inequality that represents the situation.

Solution

<table>
<thead>
<tr>
<th>Operations in the Inequality</th>
<th>To Solve</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. First (x) is (\underline{\underline{\text{(\underline{\underline{\text{(20x + 40} \leq 100)}})}}}) from both sides of the inequality.</td>
<td>1. First (\underline{\underline{\text{(\underline{\underline{\text{(20x + 40} \leq 100)}})}}}) from both sides of the inequality.</td>
<td>(20x + 40 \leq 100)</td>
</tr>
<tr>
<td>2. Then, (\underline{\underline{\text{(\underline{\underline{\text{(20x + 40} \leq 100)}})}}})</td>
<td>2. Then (\underline{\underline{\text{(\underline{\underline{\text{(20x + 40} \leq 100)}})}}}) both sides by (\underline{\underline{\text{(\underline{\underline{\text{(20)}}})}}}).</td>
<td>(\underline{\underline{\text{(\underline{\underline{\text{(20x + 40} \leq 100)}})}}}) (\underline{\underline{\text{(\underline{\underline{\text{(20)}}})}}}) (\underline{\underline{\text{(\underline{\underline{\text{(60)}}})}}})</td>
</tr>
</tbody>
</table>

Cathy can buy \(\underline{\underline{\text{\(\underline{\underline{\text{\(x \leq 3\)}}}\)}}}\) sweaters.

Since it \(\underline{\underline{\text{\(\underline{\underline{\text{\(is\)}}}\)}}}\) possible to buy a negative number of sweaters, a graph of the solution set \(\underline{\underline{\text{\(\underline{\underline{\text{\(will\)}}}\)}}}\) include values less than 0. Cathy could not have bought part of a sweater so the graph is a \(\underline{\underline{\text{\(\underline{\underline{\text{\(closed\)}}}\)}}}\).

B  Graph the solution set.

TRY THIS!

3a. A CD costs $12 and a DVD costs $15. You have $60. You plan to buy 1 DVD and some CDs. Write and solve an inequality to determine how many CDs you can buy. What does the solution mean in this situation?
REFLECT

3b. Would you use a ray or a set of points for this solution? What is the solution set? Explain.

PRACTICE

Solve each inequality. Round to the nearest hundredth, if necessary.

1. \[10x + 4 \geq -6\]
2. \[-3x - 21 > 16\]
3. \[\frac{x}{2} + 1 \geq 4\frac{1}{2}\]
4. \[\frac{x}{5} + 11 < 15\]
5. \[1.5x - 2 \leq 16\]
6. \[0.2 > -1.2x - 5.1\]

Solve each inequality. Then graph the solution set.

7. \[-5x - 17 \leq 38\]
8. \[42 < -\frac{y}{9} + 30\]

9. Dominique has $5.00. Bagels cost $0.60 each and a small container of cream cheese costs $1.50.
   a. How many bagels can Dominique buy if she also buys one small container of cream cheese? Explain your answer.
   b. Graph the solution set.

Yasmine and Alex each have $200 to spend on clothes. Use the table for 10–11.

10. Yasmine decides to purchase a jacket and some long-sleeve shirts. How many long-sleeve shirts can she buy?

<table>
<thead>
<tr>
<th>Item</th>
<th>Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-sleeve shirt</td>
<td>15</td>
</tr>
<tr>
<td>Long-sleeve shirt</td>
<td>20</td>
</tr>
<tr>
<td>Pair of jeans</td>
<td>30</td>
</tr>
<tr>
<td>Jacket</td>
<td>50</td>
</tr>
</tbody>
</table>

11. Alex wants to buy a jacket, 2 long-sleeve shirts, and some short-sleeve shirts. Can she buy at least 8 short-sleeve shirts? Explain.